Modified Euler's method

Formula:

$$y_{n+1} = y_n + h\left[f\left(x_n + h/2,\ y_n + \frac{h}{2} f(x_n, y_n)\right)\right]$$

Prob: 1

1. Compute y at $x = 0.25$ by Modified Euler method given $y' = 2xy$, $y(0) = 1$

Soln: Given $f(x, y) = 2xy$, $y(0) = 1$

Here $x_0 = 0$, $y_0 = 1$

$h = 0.25$ $x_1 = x_0 + h = 0.25$.

By Modified Euler's method.

$$y_1 = y_0 + h\left[f\left(x_0 + h/2,\ y_0 + \frac{h}{2} f(x_0, y_0)\right)\right]$$

$$y_1 = 1 + 0.25\left[f\left(0 + \frac{0.25}{2},\ 1 + \frac{0.25}{2} f(0, 1)\right)\right]$$

$f(0,1) = 2 \times 0 \times 1 = 0$

$$y_1 = 1 + 0.25\,[f(0.125, 1+0)]$$

$$y_1 = 1 + 0.25\,[f(0.125, 1)]$$

$$y_1 = 1 + 0.25\,(2(0.125)(1))$$

$$y_1 = 1 + 0.25(0.25) = 1.0625$$

$$\boxed{y_1 = y(0.25) = 1.0625}$$

Prob: 2 Find $y(0.1)$ & $y(0.2)$ using Modified Euler's method Given that

$$\frac{dy}{dx} = 1 - y, \quad y(0) = 0$$

Soln: $f(x, y) = 1 - y$, $y(0) = 0$.

$x_0 = 0$, $y_0 = 0$, $h = 0.1$ ✓

W.K.T $x_1 = x_0 + h = 0 + 0.1 = 0.1$

$$y_1 = y_0 + h\left[f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2} f(x_0, y_0)\right)\right]$$

$$y_1 = 0 + 0.1\left[f\left(0 + \frac{0.1}{2}, 0 + \frac{0.1}{2} f(0,0)\right)\right]$$

$f(0,0) = 1 - 0 = 1$ ✓

$$y_1 = 0.1\,[f(0.05, 0.05 \times 1)]$$

$= 0.1\left[1 - 0.05\right] = (0.1)(0.95)$

$\boxed{y_1 = y(0.1) = 0.095}$ ✓

$y_2 = y_1 + h\left[f\left(x_1 + \frac{h}{2}, y_1 + \frac{h}{2} f(x_1, y_1)\right)\right]$

$y_2 = 0.095 + 0.1\left[f\left(0.1 + \frac{0.1}{2}, 0.095 + \frac{0.1}{2} f(0.1, 0.095)\right)\right]$

$f(0.1, 0.095) = 1 - 0.095 = 0.905$

$y_2 = 0.095 + 0.1\left[f(0.15, 0.095 + 0.05 \times 0.905)\right]$

$y_2 = 0.095 + 0.1\left[f(0.15, 0.14025)\right]$

$f(0.15, 0.14025) = 1 - 0.14025$

$= 0.85975$

$\therefore y_2 = 0.095 + (0.1)(0.85975)$

$y(0.2) = 0.18098$

Hence $y(0.1) = 0.095$ &

$y(0.2) = 0.18098$

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